Satellite Selection for the Global Positioning System

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The selection of the optimum four satellites to use in global positioning system navigation results in a substantial computational load. This paper discusses techniques for reducing this computational load by choosing sets of satellites which, although not always the optimum, are adequate for the job. A suboptimal choice of five satellites in low-cost sequential systems is proposed and shown to usually have better performance than the optimally chosen four satellites, even though chosen with a greatly reduced number of computations.

Introduction

THE Navstar Global Positioning System (GPS) is a satellite navigation system that, when fully operational, will have 18 spacecraft in six orbital planes.[1,2] The GPS uses two pseudorandom codes in making the timing measurements. The P code provides for precision measurement of time and will be classified and not available to the general user. The C/A (clear/acquisition) code provides for easy lock-on to the signals and is available to the general user. While not allowing for the accuracy of the P code, the C/A code can be used for navigation. Precise data handling also involves four receivers operating in parallel, each receiver tracking a different satellite. For low-cost applications of the GPS technology, a single receiver, sequencing through the satellites, can be used.

A substantial computational burden exists in the application of the GPS for navigation. Reducing the number of computations in selecting the set of satellites to track is the area investigated by this paper. The next section contains a discussion of the satellite selection problem. That section is followed by one in which simplified selection algorithms are investigated. Finally, a proposal to use five satellites instead of four in sequential systems is introduced.

Satellite Selection

The original GPS design called for 24 satellites, in which case up to 11 satellites could have been visible at one time, that is, more than 5 deg above the horizon. With the present design using 18 satellites, up to eight satellites can be in view at a given time. This number is the result of a study using the Keplerian parameter representation for the GPS spacecraft ephemerides.[3] A set of ephemeris representation parameters (referred to as an almanac) was developed for 18 satellites in six orbit planes. The orbit planes are equally spaced around the equator, with inclinations of nearly 55 deg, zero eccentricity, and satellites displaced by 40 deg from one orbit to the next. Tests of this almanac at 12-min intervals for 24 h, with the user located in a fixed location at the GPS receiver antenna at the author's laboratory, showed that at most eight satellites are in view.

The amount of computation involved in satellite selection is determined by the number of satellites in view. If n satellites are in view, and four are needed to navigate, then $n!/[4!(n-4)!]$ subsets of four satellites exist. The normal measure of the goodness of a particular set of satellites is the geometric dilution of precision (GDOP).[4] The computation of the GDOP for a particular set of satellites requires the inversion of a 4×4 matrix. If eight satellites are in view, then 70 matrix inversions must be computed just to determine which set has the best (smallest) GDOP at that time. These computations would have to be repeated at regular intervals when navigating. For this reason, simplified satellite selection procedures are desired.

The goal of the simplified satellite selection procedure is to pick a set of satellites with a reasonable GDOP, and to do this with minimal computation. In order to justify the simplified or suboptimal techniques, a more detailed explanation of the problem is required. The user and satellite positions are represented in an earth-center-earth-fixed (ECEF) corrdinate system. The pseudorange measurement consists of the actual range from the user to the satellite, plus an offset due to the bias in the user's clock, plus noise. Assuming that the satellite positions are known, four measurements result in four nonlinear equations for the user position coordinates (x, y, and z) and clock bias b.

$$r_{mi}=r_i+n_i=[(x-x_i)^2+(y-y_i)^2+(z-z_i)^2]^{1/2}+b+n_i \quad (1)$$

where the subscripted variables refer to the coordinates of the i-th satellite.

Linearizing these equations with respect to the four unknowns results in a set of four linear equations, whose coefficients can be written as a 4×4 matrix. Each row of this matrix consists of the three direction cosines for the vector from the user to each satellite plus, in the fourth column, the derivative of the bias with respect to the bias, which is one. This matrix is denoted by H, and a vector containing the three direction cosines is called the unit vector.

$$H=\begin{bmatrix}(x-x_1)/(r_1-b) & (y-y_1)/(r_1-b) & (z-z_1)/(r_1-b) & 1\\ \vdots & \vdots & \vdots & \vdots\\ (x-x_4)/(r_4-b) & (y-y_4)/(r_4-b) & (z-z_4)/(r_4-b) & 1\end{bmatrix} \quad (2)$$

The GDOP computation is as follows.

$$\mathrm{GDOP}=(\mathrm{Trace}(H^TH)^{-1})^{1/2} \quad (3)$$

(The square root of the sum of the squares of the elements of H^{-1} gives the same result with less computation if H is square.)

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One condition that results in a poor GDOP is if two satellites are close together. Then two rows of H are nearly identical and H is nearly singular. This leads to the idea that the ideal case is the one where the vectors from the user to the satellites are orthogonal. Unfortunately, there are four vectors, and only three of these can be orthogonal. Another related geometric idea is the correlation between low GDOP and the high volume of the tetrahedron formed by the points of the unit vectors from the user to the satellites.[4] While this idea is not used directly, it did motivate some of the work discussed here. It also leads to an explanation of other configurations of satellites that have large GDOPs. If the four satellites and the user are nearly in the same plane, then the volume mentioned above is small and the CDOP is poor.

Algorithms

The problem of reducing the computational burden in satellite selection has been discussed in the literature. The optimum solution is to compute the GDOP for all combinations of satellites and pick the combination with the minimum GDOP. Fang[5] shows how to reduce the number of computations in obtaining the optimum GDOP when computing H^{-1}. This approach uses the fact that the combinations of satellites can be chosen so that only one row of H changes at a time. He also shows that the inverse of a matrix can be obtained from the inverse of one that differs from it in one row or column. While reducing the amount of computation to obtain the optimum GDOP, a substantial amount of computation remains.

Suboptimal techniques for GDOP computation have been proposed by Noe et al.[6] and Noe and Parsiani.[7] These techniques start by picking an orthogonal coordinate system, centered at the user, and finding the three satellites closest to the axes of this system. The fourth satellite can be chosen as the one that gives the minimum GDOP. This reduces the number of times that Eq. (3) must be computed to $n-3$. Another way to pick a fourth satellite is to sum the vectors from the user to the first three satellites and pick the fourth satellite as the one closest to the negative of this resultant.[7] This suboptimal approach was shown to be more than 100 times faster than the optimum computation and gave reliable results when tested with a 24 satellite Navstar system.

The suboptimum algorithm proposed by this paper does not start by picking a local orthogonal axis system, but it generates one for the particular set of satellites in view. This is a more consistent approach than starting with axis systems in fixed directions or that depend upon the direction of travel of the user, as is the case with the previous results.

The first step in all suboptimal algorithms is arbitrary. This step involves which satellite(s) to pick first. The previous work started by picking an orthogonal local coordinate system and finding the satellites closest to the axes. The approach used here is to start with the satellite that has the maximum elevation on the grounds that this one is usually in the optimum set. Starting with this satellite, the algorithm is based on the Gram-Schmidt orthonormalization procedure. However, the satellites are picked and the coordinate system axes are generated recursively. The complete orthonormalization algorithm is as follows:

1) Use the coordinates of the satellite with the maximum elevation to compute the first row of the H matrix, h_1, and normalize it to obtain a vector u_1.

$$u_1 = h_1 / |h_1| = h_1/\sqrt{2} \tag{4}$$

2) Pick the satellite whose row vector has the smallest dot product with the row vector of the first satellite picked. Let the row vector of this satellite be h_2. Then use the Gram-Schmidt orthonormalization procedure to produce a vector orthogonal to the first vector. Compute

$$v_2 = h_2 - [(u_1 \cdot h_2)u_1] = h_2 - [(h_1 \cdot h_2)h_1/2] \tag{5}$$

$$u_2 = v_2 / |v_2| \tag{6}$$

3) Pick the third satellite as the one with the most orthogonal row vector to the two orthogonal vectors selected in steps 1 and 2. This can be done by finding the row vector h_3 to minimize the quantity

$$E = |[(u_1 \cdot h_3)u_1] + [(u_2 \cdot h_3)u_2]|$$

or

$$E = |[(h_1 \cdot h_3)h_1/2] + [(v_2 \cdot h_3)v_2/|v_2|^2]| \tag{7}$$

Compute v_3 and define u_3 as follows:

$$v_3 = h_3 - [(u_1 \cdot h_3)u_1] - [(u_2 \cdot h_2)u_2]$$

or

$$v_3 = h_3 - [(h_1 \cdot h_3)h_1/2] - [(v_2 \cdot h_2)v_3/|v_3|^2] \tag{8}$$

$$u_3 = v_3 / |v_3| \tag{9}$$

4) Find the row vector h_4 so that the following is a minimum.

$$E = |[(u_1 \cdot h_4)u_1] + [(u_2 \cdot h_4)u_2] + [(u_3 \cdot h_4)u_3]|$$

or

$$E = |[(h_1 \cdot h_4)h_1/2] + [(v_2 \cdot h_4)v_2/|v_2|^2] + [(v_3 \cdot h_4)v_3/|v_3|^2]| \tag{10}$$

Note that it is not necessary to compute any of the normalized vectors u_i.

Tests of this algorithm showed that it is reliable, that is, it picks sets of satellites with GDOPs close to or the same as the optimal set. For example, selecting a new set every 12 min for 24 h gave the following results. The optimal and suboptimal sets of satellites are the same 64 out of 121 times. When the sets are different, the GDOP of the suboptimal set differs by more than one from the GDOP of the optimal set ten times. At no times do the two GDOPs differ by more than two.

The algorithm described above becomes more complicated as each successive satellite is chosen. For this reason, a modification to the algorithm has been made to reduce the amount of computations involved in picking the last satellite. This modification is motivated by the idea of relating the GDOP to a volume that is mentioned in a prior section. The method starts by finding the equation of the plane formed by the points of the unit vectors of the first three satellites chosen. The fourth satellite is chosen as the one whose unit vector point is the maximum distance from that plane.

The equation of the plane is

$$ax + by + cz + d = 0 \tag{11}$$

where

$$a = (y_2 - y_1)(z_3 - z_1) - (y_3 - y_1)(z_2 - z_1)$$

$$b = (x_3 - x_1)(z_2 - z_1) - (x_2 - x_1)(z_3 - z_1)$$

$$c = (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)$$

$$d = -ax_1 - by_1 - cz_1 \tag{12}$$

and x_i, y_i, z_i are the coordinates of the satellites. Given a point off the plane with coordinates u, v, w, the distance to the plane is[8]

$$D = \left| \frac{au + bv + cw + d}{(a^2 + b^2 + c^2)^{1/2}} \right| \tag{13}$$

Since the denominator of this expression is constant for a particular computation cycle, only the numerator is used in picking the fourth satellite.

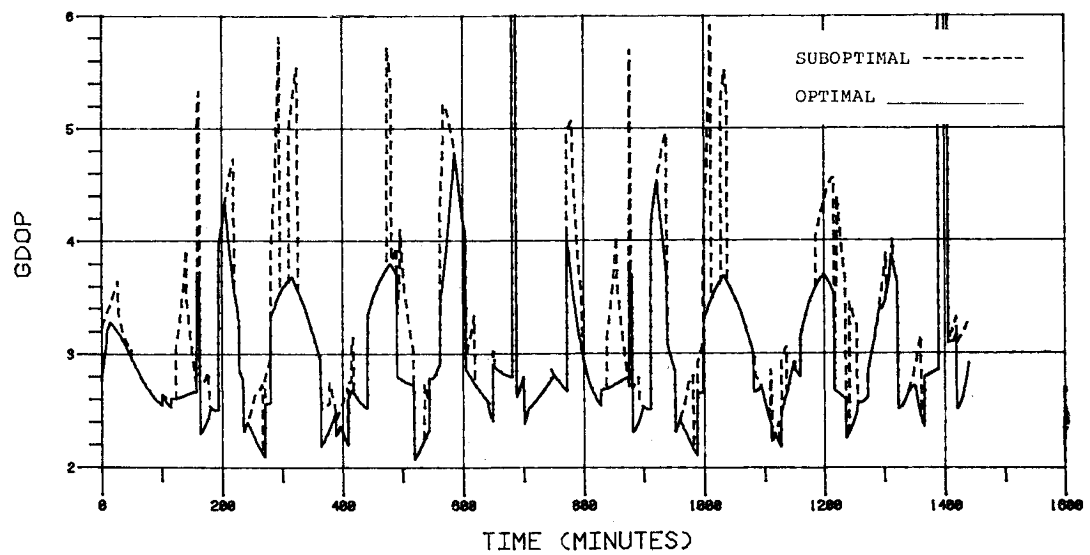

Fig. 1 Optimal and suboptimal GDOP vs time using four satellites. GDOP computed at 1-min intervals for 24 h.

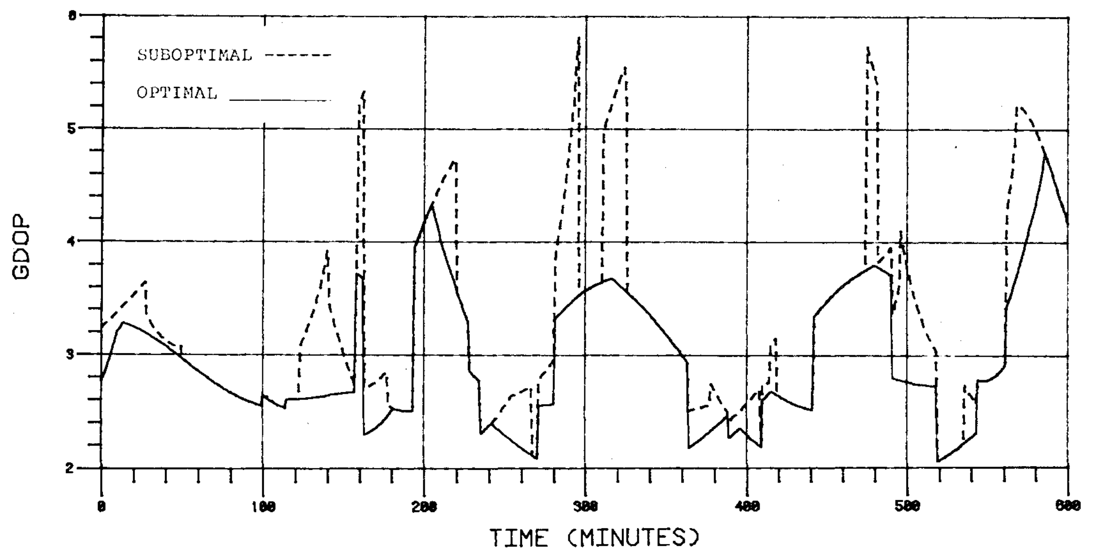

Fig. 2 Expansion of Fig. 1 showing the first 10 h.

The computational savings of the maximum distance technique are substantial. Given that three satellites have been chosen, the Gram-Schmidt approach requires $12n$-32 multiplications and divisions to pick the fourth satellite. The computations replaced by the maximum distance approach are the ones for v_3 in step 3 and all of step 4. The new step 4 is the maximum distance computation which just requires $3n$ multiplications. The amount of computation is also much less than picking the satellite nearest the negative resultant vector of the first three satellites.

The results of using this modification are identical to using the complete orthonormalization algorithm for the cases that have been tested. The maximum distance computation can also be used with the actual satellite and user positions, rather than the unit vector positions. This case has been tested and gave results which only differed at a few instants of time from the unit vector case, with no dramatic differences between the two cases. The unit vector case is recommended because it uses the elements of the H matrix, which are used in the actual GDOP computation.

In order to better display the results of the satellite selection algorithm, plotting routines have been incorporated into the simulation. Figure 1 shows the plot of the GDOP for the set of satellites selected by the algorithm that uses the orthonormalization procedure on the rows of the H matrix to select three satellites and the maximum distance criterion on the unit vectors of the remaining satellites to pick the fourth. The data for this plot are computed at 1-min intervals for 24 h. The dashed line is the GDOP of the suboptimal set. The two occasions when the plot goes off scale are when only four satellites are in view. Figure 2 shows an expansion of the first 10 hours of the data. The discontinuities in the plots are caused by changes in GDOP due to satellites just entering or leaving view.

One other algorithm was tested using the distance computations. In this case, the third satellite is chosen as the one furthest from the plane containing the points of the unit vectors of the first two satellites plus the user. The fourth one is chosen using the distance algorithm as above. This procedure is not as reliable as the recommended algorithm, since it occasionally produces sets of satellites with GDOPs several times larger than the optimal.

Using Five Satellites

The number of satellites used for navigation has, thus far, been assumed to be four. However, if the pseudorange measurements are made sequentially, there is no reason why

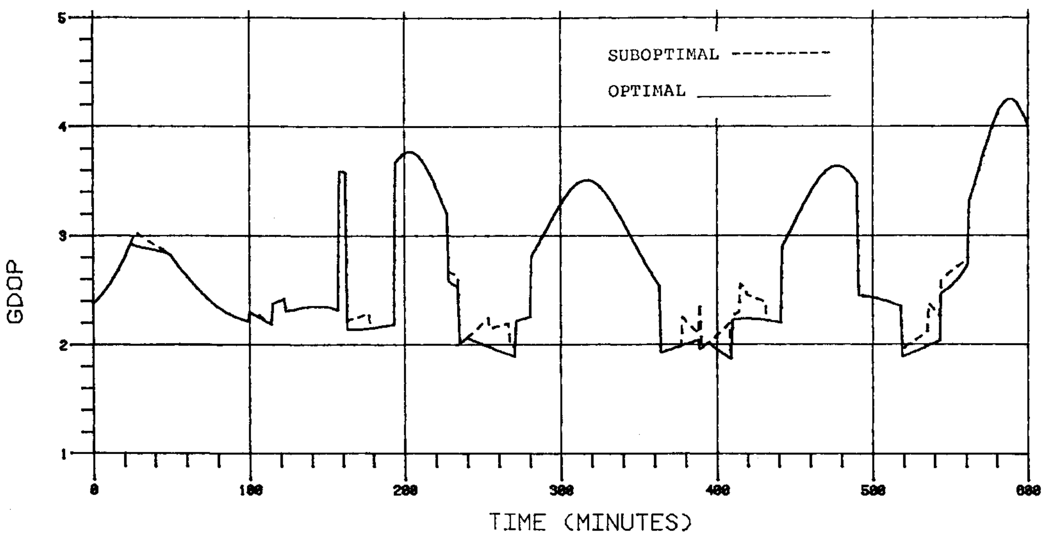

Fig. 3 Optimal and suboptimal GDOP vs time using five satellites. GDOP computed at 1-min intervals for 10 h.

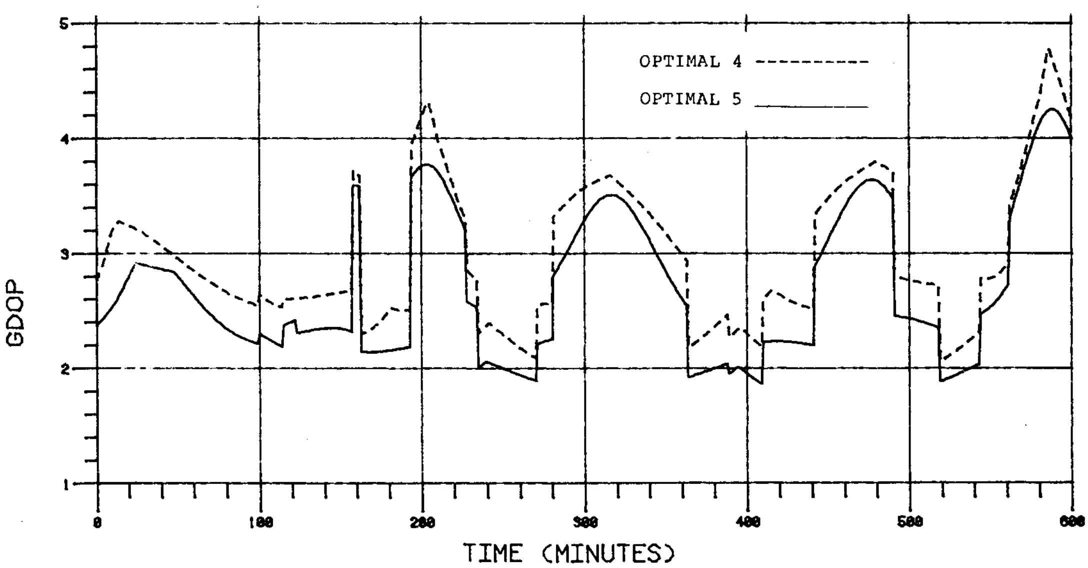

Fig. 4 Optimal GDOP vs time using four and five satellites.

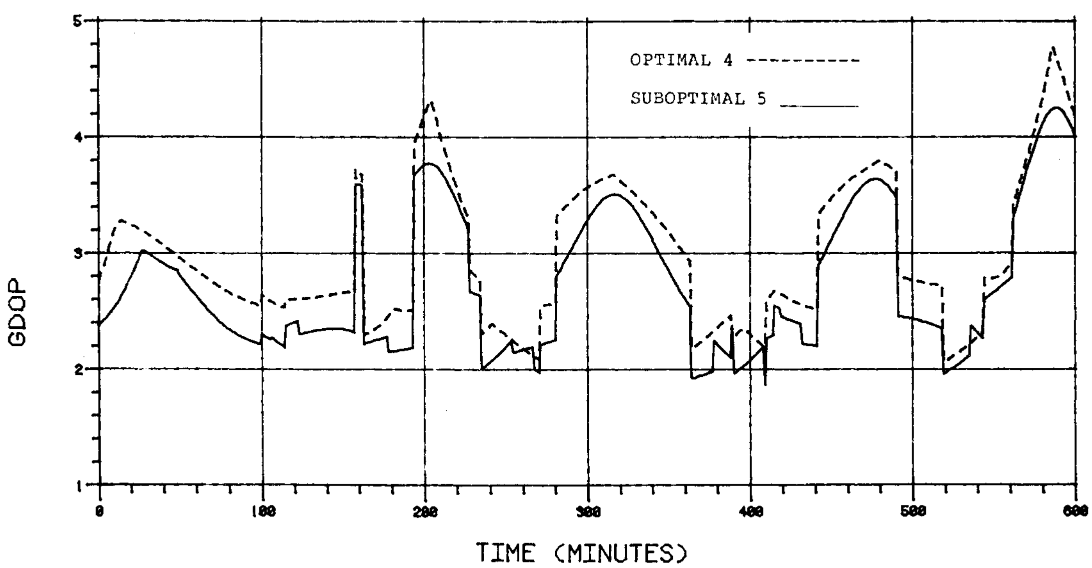

Fig. 5 Optimal GDOP for four satellites and suboptimal GDOP for five satellites vs time.

more than four satellites cannot be used. Since there is a great deal of time when only five satellites are in view, the use of five for navigating obviates the need for a selection procedure at those times. Even more important is the fact that the GDOP is reduced by adding another satellite to a set of four. If n measurements are being used, H is an $n \times 4$ matrix; but H transposed times H is still 4×4. The definition of GDOP is still valid in this case.[1] For this reason, the idea of using five satellites and a selection algorithm for five satellites is worthy of further investigation. However, since H is not square, the shortcut approaches of computing GDOP directly from H^{-1} cannot be used.

Fortunately, the algorithm recommended for the selection of four satellites can be modified for the selection of five satellites at the cost of almost no additional computation. The last step in the distance computation is to take the absolute value of a signed number which represents the distance above or below the plane. By working with the signed number and taking the satellites that produce the largest and smallest values, the result is usually the points with the greatest distance above and below the plane. The test run of 12-min intervals for 24 h always showed satellites on both sides of the plane when six or more satellites are in view. The selection algorithm picked the set with the minimum GDOP 92 out of 121 times, and the GDOP of the suboptimal set is always within one of the optimal. These statistics are somewhat deceiving in that there are less than six satellites in view 49 of these times. Figure 3 shows the plots of the GDOPs for the optimal and suboptimal satellite sets, where the data are generated at 1-min intervals for 10 h.

Another five-satellite selection procedure was tested and rejected because it was more complicated and did not perform as well as the above algorithm. This case started with the recommended procedure for picking four satellites. Then the dot products of the remaining vectors with all the chosen vectors are computed, and the vector is chosen whose maximum dot product is a minimum.

A comparison of the five satellite results with the four satellite results is useful. Figure 4 shows the optimal GDOPs for sets of both four and five satellites. The dashed line is the four satellite case. The GDOP of the five satellite set is always less than that of the four satellite set. Figure 5 is even more interesting. It shows that the GDOP of the suboptimal set of five satellites is usually less than the GDOP of the optimal set of four satellites. This result indicates that an advantage may exist to using five satellites for navigation when making sequential measurements. This advantage was confirmed in the simulations where it was shown that five satellite navigation be achieved with no increase in filter computational burden and only a slight increase in the amount of memory used.

To summarize the advantage of the suboptimal algorithms, the number of multiplications and divisions required to optimally choose four and five satellites, when eight are visible, is 5600 and 8000, respectively. The suboptimal approach requires just 149 multiplications and divisions for both cases. The number of computations to obtain the optimal GDOP can be reduced by using the approach described by Fang,[5] but the reduced number of computations is much greater than the suboptimal numbers. These results clearly indicate the computational savings achieved by the suboptimal techniques.

Conclusions and Recommendations

This paper has presented several new ideas for selecting a set of satellites to use for navigating with the Navstar system. These ideas are especially useful for low-cost systems that can be implemented with a single channel receiver and a single microprocessor for signal processing and filtering. A suboptimum satellite selection algorithm has been described which requires an amount of computation which is two orders of magnitude less than that required to pick the optimum set. This algorithm is reliable in that it never picked a set of satellites with a GDOP that differed significantly from the optimum. The algorithm was tested with the number and orientation of satellites which is in the current GPS design.

This paper also introduces the idea of using five satellites for low-cost sequential GPS systems. This often obviates the selection problem since only five satellites may be in view. The algorithm to pick five satellites requires no more computation than the one to pick four. The geometric dilution of position of the suboptimal five satellites is usually better than that of the optimal four. The idea of navigating with five satellites has been tested by means of a simulation and found to work well with no increase in the filtering computational time burden.

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